Filtered manifolds with distinguished transformations and transformations groups

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Commentary

My work comprises the study of geometries with distinguished transformations and properties of the geometries following from the existence of these transformations. In my research, I focus mostly on parabolic geometries that form important class of geometric structures and are extensively studied in last years, [ČaSl09]. The thesis consists of papers [Zal10b, GrZa17, GrZa18, KrWiZa18, HrZa19].

Main part of the work is based on collaboration with Jan Gregorovič devoted to various generalizations of symmetric spaces. We combine several methods for studying of parabolic geometries. My viewpoint is mostly geometric and is based on studying of Weyl connections, their relation to curvatures and their compatibility with transformations, [Zal09, Zal10a] and [Zal10b]. The viewpoint of Jan Gregorovič is based on studying of homogeneous geometries and their description via functorial constructions based on algebraic methods, [Gre12c, Gre12a, Gre13]. The combination of the methods allows to give complete description of (both local and global) geometric properties of generalized symmetric geometries and their classification, [GrZa16b, GrZa15b] and [GrZa17]. As an example we present generalized symmetries of almost CR structures, [GrZa18].

The next part of the work concerns submaximally symmetric parabolic geometries. Complex submaximally symmetric parabolic geometries are completely described in [KrTh14], and it follows from the discussion that real submaximally symmetric parabolic geometries shall be studied case by case. In the collaboration with Boris Kruglikov and Henrik Winther we study submaximally symmetric almost quaternionic structures, which are real parabolic geometries. We give submaximal dimensions and models in [KrWiZa18].

Finally, there are interesting applications in geometric methods in control theory. In my collaboration with Jaroslav Hrdina, we focus on control theory on Lie groups. For various non–holonomic mechanisms, their configuration spaces are filtered manifolds that often form parabolic geometries modeled on nilpotent Lie groups. We study controllability and optimal control of the mechanisms with filtration $(4, 7)$ using properties of these geometries and their transformations, [HrZa19].

As is usual in theoretical mathematics, all results were developed by uniform collaboration of all authors, i.e. each author participated on each part of the work. Thus the author contribution is 50% in the case of two authors and 33% in the case of three authors.

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